كنتُ أعلى علامة على الشعبة في ذلك الفصل

احتمالات ومتغيرات عشوائية - امتحان ثاني

Specialization:	Electrical Engineering, Communication Eng.		Palestinian National Authority Ministry Education & Higher Education Palestine Technical University College of Engineering & Technology
Course Name:	Probability and Random Variables		
Date:	11/04/2011		
Time:	11:00-12:00		
Instructor:	Dr. Mutamed Khatib		Second Exam Second semester 2010/2011

Name: [redacted] معن سلام وقران Section: احمد شلالدة خميس (٨-٩) Mark: 24 /30

Please answer ALL the following FIVE questions (TEN branches)

Q.1 A factory can produce two types of transistors, Type 1 and Type 2. Let X be a random variable denoting the number of Type 1 transistors produced on a given day, and let Y be the number of Type 2 transistors produced on the same day. The joint probability mass function for X and Y is given by

	$p_{X,Y}(x,y)$	Y = 0	Y = 1	Y = 2	Y = 3
X	1	0.1	0.2	0.1	0.1
	2		0.2	0.2	
	3		0.05	0.05	

a. *(3 marks)* Compute $P(X \le 2, Y \ne 2)$

$$= P_{X,Y}(1,0) + P_{X,Y}(1,1) + P_{X,Y}(1,3) + P_{X,Y}(2,0) + P_{X,Y}(2,1) + P_{X,Y}(2,3)$$

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$$= 0.1 + 0.2 + 0.1 + 0 + 0.2 + 0 = \cancel{0.6} \quad \boxed{0.6}$$

تم الرفع بواسطة م. معن أبو عيسى

b. *(3 marks)* Compute $P(Y > 0)$

$$P(Y>0) = P(Y=1) + P(Y=2) + P(Y=3)$$

$$= (0.2 + 0.2 + 0.05) + (0.1 + 0.2 + 0.05) + (0.1)$$

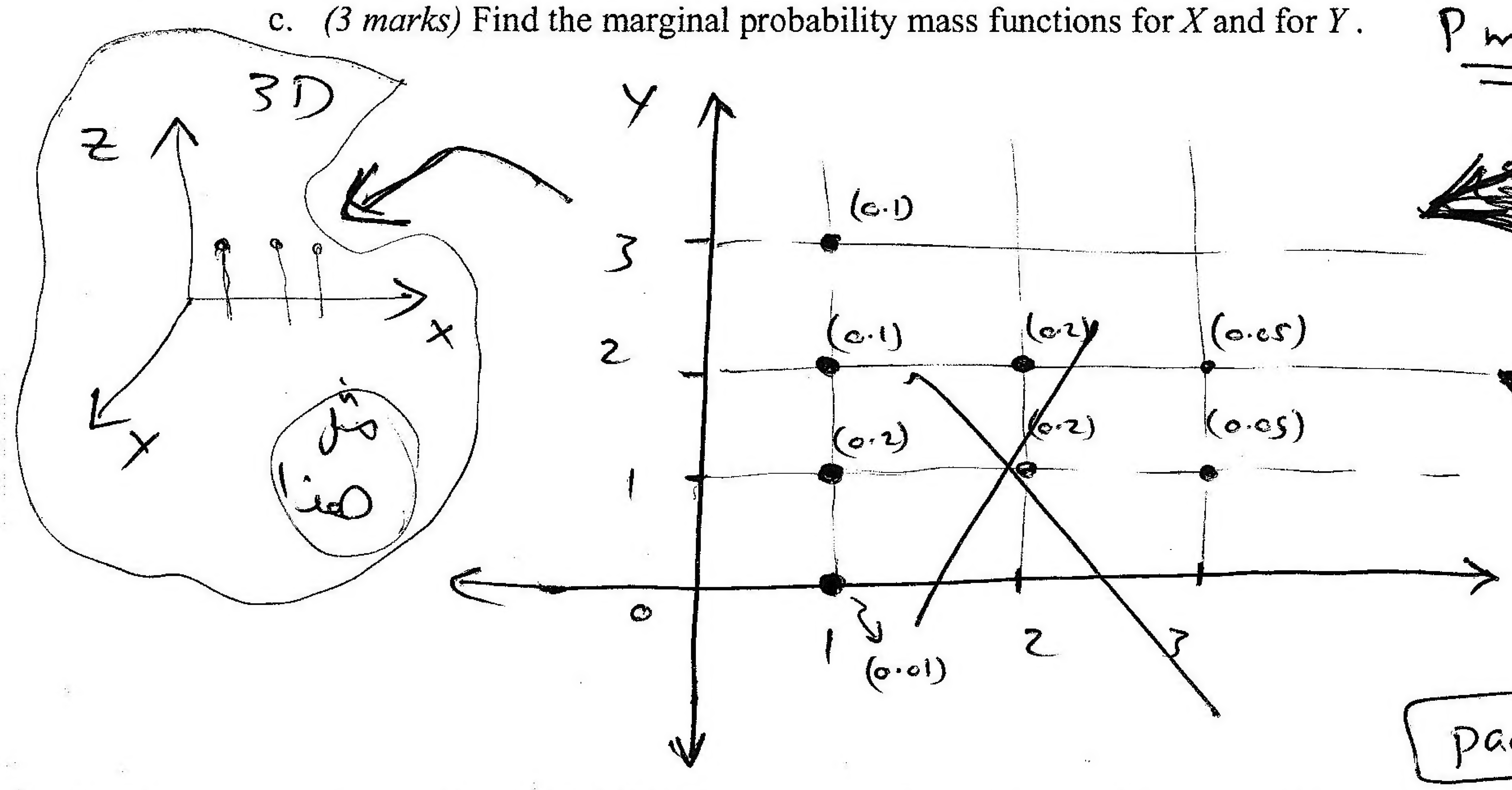
$$0.45 + 0.35 + 0.10$$

3

$$= \boxed{0.9}$$

c. *(3 marks)* Find the marginal probability mass functions for X and for Y.

Pmf



النقط عبارة عن أسهم خارجة باتجاه +ve Z axis

Q2. Let X be a continuous random variable with probability density function

$$f_X(x)=\begin{cases}1+x & -1\le x\le 0\\ 1-x & 0\le x\le 1\\ 0 & elsewhere\end{cases}$$

Pdf

a. *(3 marks)* Find $P(X\le 0.5)$

$$= \int_{-1}^{0} f_X(x)\,dx + \int_{0}^{\frac{1}{2}} f_X(x)\,dx$$

$$= \int_{-1}^{0}(1+x)\,dx + \int_{0}^{\frac{1}{2}}(1-x)\,dx = \left(x+\frac{x^2}{2}\right)\Big]_{-1}^{0} + \left(x-\frac{x^2}{2}\right)\Big]_{0}^{\frac{1}{2}}$$

3

$$= 0 - \left(-1+\frac{(-1)^2}{2}\right) + \left(\frac{1}{2}-\frac{(\frac{1}{2})^2}{2}\right) - 0$$

$$1-\frac{1}{2}+\frac{1}{2}-0.125 = 1-0.125 = \boxed{0.875}$$

b. *(3 marks)* Find $P(X^2\le 0.25)$

$$P(X^2\le 0.25) = P(|X|<\tfrac{1}{2})$$

$$= P(-\tfrac{1}{2}\le X\le \tfrac{1}{2})$$

$$= \int_{-\frac{1}{2}}^{0} f_X(x)\,dx + \int_{0}^{\frac{1}{2}} f_X(x)\,dx$$

$$= \int_{-\frac{1}{2}}^{0}(1+x)\,dx + \int_{0}^{\frac{1}{2}}(1-x)\,dx$$

3

$$= \left(x+\frac{x^2}{2}\right)\Big|_{-\frac{1}{2}}^{0} + \left(x-\frac{x^2}{2}\right)\Big|_{0}^{\frac{1}{2}}$$

$$= 0-\left(-\frac{1}{2}+\frac{(\frac{-1}{2})^2}{2}\right) + \left(\frac{1}{2}-\frac{(\frac{1}{2})^2}{2}\right) - 0$$

$$= \frac{1}{2}-\frac{1}{8}+\frac{1}{2}-\frac{1}{8}$$

$$= 0.5-0.125+0.5-0.125$$

$$= \boxed{0.75}$$

Q.3 *(3 marks)* The reading of a water level meter is a continuous random variable X with probability density function:

$$f_X(x)=\begin{cases}0.25 & -2\le x\le 2\\ 0 & elsewhere\end{cases}$$

Find Var(X)

$$Var(X) = E[(X-\mu_X)^2]$$

$$\mu(x) = \int_{-2}^{2} x\, f_X(x)\, dx = \int_{-2}^{2} 0.25X = \frac{0.25X^2}{2}\Big|_{-2}^{2}$$

$$= 0.125X^2\Big|_{-2}^{2} = 0.125(2)^2 - 0.125(-2)^2 = \boxed{0}$$

$$\therefore Var(x) = E[(X-0)^2] = E[X^2]$$

$$E[X^2] = \int_{-2}^{2} h(x) f_X(x)dx = \int_{-2}^{2} x^2(0.25) = \int_{-2}^{2} 0.25x^2 = \frac{0.25x^3}{3}\Big|_{-2}^{2}$$

$$\frac{0.25(2)^3}{3} - \left(\frac{0.25(-2)^3}{8}\right) = \frac{2}{3} - \left(-\frac{2}{3}\right) = \frac{2}{3}+\frac{2}{3} = \frac{4}{3} = \boxed{1.333}$$

$$\therefore Var(X) = \boxed{1.333}$$

Q4. *(3 marks)* Diameters of bearings in mass production are normally distributed with mean 0.25 inch and standard deviation 0.01 inch. Bearing specifications call for diameters of 0.24 ± 0.02 inch. What is the probability of a defective bearing being produced?

$\mu_x = 0.25$

$\sigma_x = 0.01$

$\circledast$ $\therefore$ We need to Compute $P(0.22 \le X \le 0.26)$ First $\circledast$

$$= P(X \le 0.26) - P(X \le 0.22)$$

$$= P\left(Z \le \frac{0.26-0.25}{0.01}\right) - P\left(Z \le \frac{0.22-0.25}{0.01}\right)$$

$$= P(Z \le 1) - P(Z \le -3)$$

$$= P(Z\le 1) - (1 - P(Z \le 3))$$

$$= P(Z\le 1) - 1 + P(Z \le 3)$$

$$= \boxed{P(Z\le 1) + P(Z\le 3) - 1}$$

$\therefore$ The probability of defective bearing being produced $= \boxed{1 - P(0.22 \le X \le 0.26)}$

$$= 1 - (P(Z\le 1) + P(Z\le 3) - 1)$$

$$= \boxed{2 - P(Z\le 1) - P(Z\le 3)}$$

Q.5 *(3 marks)* Suppose that the life lengths of two electronic devices, $D1$ and $D2$, have distributions $N(40, 36)$ and $N(45, 9)$, respectively. If the electronic device is to be used for a 45hour period, which device is to be preferred? If it is to be used for a 50hour period, which device is to be preferred?

$$\boxed{\begin{array}{l} D1 \rightarrow N(40, 36) \\ D2 \rightarrow N(45, 9) \end{array}}$$

$\Rightarrow D1 \rightarrow P(X \leq 45) = P\left(Z \leq \frac{45-40}{6}\right) = \boxed{P\left(Z \leq \frac{5}{6}\right)}$

$\Rightarrow D1 \rightarrow P(X \leq 50) = P\left(Z \leq \frac{50-40}{6}\right) = P\left(Z \leq \frac{10}{6}\right) = \boxed{P\left(Z \leq \frac{5}{3}\right)}$

$\Rightarrow D2 \rightarrow P(X \leq 45) = P\left(Z \leq \frac{45-45}{3}\right) = \boxed{P(Z \leq 0)}$

$\Rightarrow D2 \rightarrow P(X \leq 50) = P\left(Z \leq \frac{50-45}{3}\right) = \boxed{P\left(Z \leq \frac{5}{3}\right)}$

* According to Device D1:

[1] $\boxed{P(X \leq 45) = P\left(Z \leq \frac{5}{6}\right)}$

[2] $\boxed{P(X \leq 50) = P\left(Z \leq \frac{5}{3}\right)}$

* According to Device D2:

[1] $\boxed{P(X \leq 45) = P(Z \leq 0)}$

[2] $\boxed{P(X \leq 50) = P\left(Z \leq \frac{5}{3}\right)}$

* Then, we make comparisons between these probabilities to decide what device is preferred for every case.

Q.6 Suppose (X,Y,Z) is a random vector with joint probability density function

$$f_{X,Y,Z}(x,y,z)=\begin{cases}\alpha(xyz^2) & 0\le x\le 1,\ 0\le y\le 2,\ 0\le z\le 2\\ 0 & \textit{elsewhere}\end{cases}$$

a. *(3 marks)* Find the probability that X will take on a value less than 0.5, and Y and Z will both take on values less than 1.

$$\int_{x=0}^{0.5}\int_{y=0}^{1}\int_{z=0}^{1}\alpha(xyz^2)\,dx\,dy\,dz$$

$$=\alpha\int_{y=0}^{1}\int_{z=0}^{1}\frac{x^2yz^2}{2}\Big|_0^{0.5}dy\,dz=\alpha\int_{y=0}^{1}\int_{z=0}^{1}0.125\,yz^2\,dy\,dz$$

$$=\alpha\int_{z=0}^{1}\frac{0.125\,y^2z^2}{2}\Big|_0^1 dz=\alpha\int_{z=0}^{1}0.0625\,z^2\,dz$$

$$=\alpha\left(\frac{0.0625\,z^3}{3}\right)\Big|_0^1$$

$$=\boxed{0.0208\,\alpha}$$

3

$\circledast \Longrightarrow$ $0.0208\,\alpha=1$ $\boxed{\text{CDF Properties}}$ $\therefore\ \alpha=\frac{1}{0.0208}\approx\boxed{48}$

b. *(3 marks)* Find the probability density function for the random vector (X,Y)

$$\int_{x=-\infty}^{1}\int_{x=-\infty}^{2}\int_{z=-\infty}^{\infty}\alpha(xyz^2)\,dx\,dy\,dz$$

$$=\int_{x=0}^{1}\int_{y=0}^{1}\alpha(xyz^2)\,dx\,dy=\alpha\int_{y=0}^{1}\frac{x^2yz^2}{2}\Big|_0^1 dy$$

$$=\alpha\int_{y=0}^{1}\frac{yz^2}{2}\,dy=\alpha\left(\frac{y^2z^2}{4}\right)\Big|_0^2=\alpha\left(\frac{4z^2}{4}\right)$$

$$=\boxed{\alpha z^2}$$

$$=\boxed{48z^2}$$

Good luck